Journal of Management and Science
12(3) 2022 144-148
OPEN ACCESS
RESEARCH ARTICLE
DOI: https://doi.org/10.26524/jms.12.77

Morphisms and Economic Modeling

Diego Lanzi [1]

Abstract

Morphisms are widely used in several branches of scienti c inquiry, but not so much in economics. Nevertheless, many conceptual advan- tages in the economic modeling from using a categorical setting exist. In this paper, we discuss why morphisms could be successfully injected into economic modeling and, in particular, that allomorphisms, i.e., structure- altering maps that re-shape economic processes, can be useful for letting economics to be part of relational social science.

Keywords: Category theory, Morphism, Economic Modelling, Relational Social Science.

Author Affiliation: [1] University of Bologna.

Corresponding Author: Diego Lanzi. University of Bologna.

Email: diego.lanzi@unibo.it

How to cite this article: Diego Lanzi. Morphisms and Economic Modeling, Journal of Management and Science, 12(3) 2022 144-148. Retrieved from https://jmseleyon.com/index.php/jms/article/view/591

Received: 26 March 2022 **Revised:** 1 April 2021 **Accepted:** 4 April 2022

1. Introduction

The term morphism comes from the ancient Greek's word morph , i.e., form or shape, and it expresses the state of having a speci ed shape. The concept is widely used in several branches of scienti c inquiry from category theory and topology, to biology, semiotics, linguistics and computer science. Surprisingly, the idea of morphism does not nd large application in economics. This surely not because forms and shapes do not enter in the economic discourse. Choice sets, utility scales, game forms and economic mechanisms can take di erent forms and shapes.

As stressed by Crespo and Tohme [1], recent research advances has shown that there are alternative, and more convenient, ways of representing mathemat- ical ideas than the traditional ones. Central to these advances has been category theory. Well-established consensus exists among pure and applied mathematicians that:

Category theory has come to occupy a central position in contempo- rary mathematics and theoretical computer science, and is also applied to mathematical physics [...] category theory is an alternative to set theory as a foundation for mathematics. [2]

Category theory is focuses on the relations among objects. Intuitively, while set- theoretical foundations de ne functions in terms of their domain and range sets, category theory takes functions by themselves as the elements of interest. More precisely, any category is described by the morphisms between its objects.

The use of categorical settings can yiled many conceptual advantages for economics. On one hand, category theory changes the focus from objects to morphisms. Such a focus on morphisms frees economic models from the excessive emphasis given to equilibrium which becomes something that may, or may not, exist in the appropriate category. Furthermore, the relational aspect of morphisms does not demand that every entity is dened in terms of simpler entities:objects are given without any consideration to their inner structure, and dened by their interactions with other objects. Such a change of perspective in game theory [3,4] or economic systems theory [5] has already produced interesting results. On the other hand, category theory can help in clarifying existing relations between individual and aggregate behavior that are very important in
social choice theory and welfare economics. Dierent aggregation procedures can exist in dierent categories, and how collective decision processes are categorically conceived by groups and collectives is an interesting research issue for both empirical and theoretical economics.

Finally, as we discuss in this paper, category theory can be useful in agent- based models which explicitly want to deal with the context of decision-making that emerges from the decision itself. This last class of economic models should.

The remaining of this paper is organized as follows. In Section 2 we will sketch main features of morphisms. Sections 3 relates category theory to economic modeling and introduces a coalgebraic framework. Section 4 discusses main pillars of relational social sciences, and why allomorphisms, i.e., structurealtering maps that re-shape economic processes, can be useful for making economic models consistent with such a line of scientic inquiry. Section 5 discusses two relevant types of allomorphism from which to start from, then the last Section concludes.

2. The Concept of Morphism

Category theory stipulates that the only knowledge we can have in an object is in how it relates to other objects1 .For instance, the only way to do determine if two objects are the same is to nd a morphism with special properties between them. Consistently, the strongest kind of morphism is an isomorphism. An isomorphism establishes that two objects in a category are the same object. An isomorphism admits a two-sided inverse, i.e., there is another morphism, in the category at hand, such that their compositions emit identity on the domain and codomain, respectively. Iso stands for equal in the sense that if an isomorphism exists, there is an sort of sameness to the two objects.

For an introduction to morphisms see Mac Lane [6] or Awodey. [7]

A morphism from an object onto itself that doesn't necessarily establish an identity between the two elements is named an endomorphism. "Endo" stands for "within" or "inner", and morphisms of this kind map a structure into itself. An endomorphism that is also an isomorphism is called automorphism. Any automorphism is, therefore, bijective.

See Lanzi [8]. For connections between morphisms and semiotics see Marshall and Freitas. [9]

Formally, a morphism $f : A \rightarrow B$ in a category K is an isomorphism if there exists a morphism $g : B\ A$ such that both ways to composef and g give the identity morphisms on the respective object.

A weaker form of morphism is an epimorphism. The epi root connotes mor- phisms mapping over the entirety of the codomain (i.e., surjectivity). Thus, $f: A \rightarrow B$ is an epimorphism if for every object X and every pair of morphisms $g,h:B \rightarrow X$ the composition $g_0\ f = h_0\ f$ implies that $g = h$. When a morphism is injective, it is called monomorphism where mono indicates that it generalizes one-to-one functions. In such a case, $f : A \rightarrow B$ is a monomorphism if for every object X and every pair of morphisms $g, h : X \rightarrow A$ the condi- tion $f_0 g = f\ h_0$ implies $g = h$. Straighforwardly, isomorphisms are bijective, one-to-one and onto[2] .

A structure-preserving map between objects $a, b \in X$, i.e., a homomor- phism f on some structure with a binary operation , will preserve the binary operator across the mapping; that is, $f(a \gtrsim b) = f(a) \gtrsim f(b)$:. If the structure is that of posets, an homomorphism is an order-preserving map, and an order relation. The pre x homos means similar , and endomorphisms are homomorphisms whose domain equals the codomain, while monomorphisms are commonly de ned as injective homomorphisms and epimorphisms as surjective homomorphisms.

A morphism that is not order- or structure-preserving can be named an allo- morphism3 . An allomorphism f will not preserve through the mapping a binary operation $\gtrsim$ over some $a, b \in X$, i.e., $f (a \gtrsim b) \lesseqgtr f (a) \gtrsim (b)$. In semiotics, the term allomorph describes the realization of phonological variations for a speci c morphene; the pre x allo means other or di erent and indicates that the mapping creates diversities and variations in the structure of X^3. Since allomor-phisms alter the structure on which they operate, injectivity and surjectivity are not necessary required.

Finally, morphisms are usually equipped with a relation called composition. If you have two morphisms, η and ϕ, their composition is de ned when the codomain of η is the domain of ϕ, and is denoted $\phi \circ \eta$. The composition satis es both an identity property, i.e., there always exists an identity morphism of the kind $id_A : A \rightarrow A$ such that $id_A \circ \eta = \eta = \eta \circ id_B$, and an associativity property, i.e., given a third morphism ϑ it is true that $\vartheta \circ (\phi \circ \eta) = (\vartheta \circ \phi) \circ \eta$.

The collection of all morphisms from a structure A to B is denoted Mor (A, B).

3. Coalgebras and Economic Models

The most general category is the category of all sets, where the objects are sets and morphisms are total functions. Other examples of categories are sets and relations, measurable spaces and measurable functions, or topological spaces and continuous functions. In what follows, we shall introduce some category- theoretical notions only in the special case of the category of sets. More precisely, the concepts of functor and coalgebra[4].

A functor F (from the category of sets to itself) is an operation assigning to each set X a new set F (X), and to each function $g : X \rightarrow Y$, a function $F (g) : F (X) \rightarrow F (Y)$ such that F (idX) = idF (X) andF (f g) = F (f) F (g), for all sets X and all functions f and g.

Then, we need to de ne an economic process. The latter can be described as a state-based transformation of inputs in outputs. With a given input value and, on the basis of the current state, an economic process produces an output value and a change in the state of the world. Alternatively, the process terminates.

Let be S the set of states, I the set of inputs, O the set of outputs and R The set of results of the process. A function

$$\pi : S \times I \rightarrow C (R + S \times O) \qquad (1)$$

Describes the process for some choice functor C. When in state $s \in S$ and given the input $i \in I$, the functor chooses a possible continuation that consists in either terminating with a result $r \in R$ or in continuing in a $s' \in S$ and producing an output value $c \in O$. Choice functors can be deterministic, i.e., inputs uniquely determine what happens, non-deterministic, i.e., given some inputs several possible continuations of the process exist, or probabilistic, i.e., the process continues randomly.

For a functor F , an F-coalgebra is a function $h : X \rightarrow F (X)$, for some set X. Consistently, we can re-write expression (1) as:

$$\pi : S \rightarrow \Pi (S) = C(R + X \times O)^I \qquad (2)$$

Then given a process π and a morphism f , the resulting process π > f applies the map f to state-output pairs and it is de ned by:

$$(\pi \quad f) (s) (c) = C (id + f) (\pi (s) (c)) \ (3)$$

Conversely, the writing f π indicates that, before each step, the map f is applied on inputs; that is:

$$(f > \pi) (s) (i) = \pi (s) (f (i)) (4)$$

Economic processes of this kind can be referred to consumer behavior, pro- duction activities or choice problems in which one agent assumes decisions. In standard microeconomics, there is a gap to ll because models are traditionally built in terms of systems of simultaneous equations and di erential equations. In opposition, agent-based models tries to link categories, computer science and economic principles[5] .

4. Economic Modeling and Relational Social Science

As we have seen, morphisms, essential building blocks of categories, can be applied to economic processes. In this Section, we argue that morphisms can be a useful tool for a relational social science (Emirbayer (1997)) into which economics can nd a place.

The relational perspective on social action and historical change can be char- acterized by comparing it with the substantialist one. According to the latter, substances of various kinds (objects, beings, essences, societies) constitute the fundamental units of inquiry. Contrarily, relational social sciences reject the idea that one can posit discrete, pregiven units as ultimate starting points of social analysis. Variable-based analysis is equally misleading: it detaches el- ements/substances from their spatio-temporal contexts, analyzing them apart from their relations with other elements within elds of mutual determination.

The set-up here introduced is a slight modification of the one suggested by Blumensath and Winschel. [10] For a similar framework see Hedges and Ghani. [11] See Tesfatsion et al. [12]

In opposition, a relational approach embeds agents, and processes, within relationships, contexts and stories which shift over time and space, and such a shifting precludes the categorical stability of action. The ontological embeddedness,or locatedness, of entities within actual situational contexts become central, and dynamic relations between units, seen as unfolding/ongoing processes rather than as static ties among inert substances, the bases of analysis.Put it roughly: substantialist thinking corresponds closely to grammatical patterns ofWestern linguistics. [14,15]

Relational social sciences, therefore, must be focused on embeddedness struc- tures, situational contexts, relational perspectives and the like. One might just as well speak here of construals, transactions or conversations; the underlying idea remains the same: the primacy of contextuality[6]. In economics, this means to consider the possibility that processes are context-dependent. [16]

In the economic modeling, context-dependency can be approached in many ways. For instance, contexts can mean di erent embedding perspectives and descriptions of processes which in uence agents' behavior, and the way they structure their economic decisions.

Perspectives, for instance, can be de ned as mappings between objects in the external world and one's internal language. [17] These mappings impose a sort of ontology though we all confront the same reality, the way we code this reality in our internal languages can be di erent for we may employ di erent map- ping relations when this happens, individuals will see di erent worlds even though, at rock bottom, it is the same external world they confront. [18]

Similarly, Kahneman and Tversky [19] have shown that choice behavior is sensitive to the description of the choice, and to the way in which alternatives are framed. Di erently describing the same economic problem modi es the psychological attitudes of the chooser and, therefore, causes a di erent economic behavior. [20] Description-dependency may be explained in terms of agent's moral and relational position, in terms of meaning conferred to di erent menus of choice or through commitments elicited by the way the problem is framed: in all cases, diverging descriptions map di erently how the economic problem is perceived and structured.

Nevertheless, Sometimes our perception of economic processes cannot fully be cashed out in terms of salience over bundles of per-determined objects. Sometimes it must be cashed out in terms of determining what the features, or properties, of these objects are in the rst place. For instance, following Dietrich and List [21], a context is not merely a subset of the universal set of options X, but a subset of X accompanied by some parameters which specify further features of the environment.

Such a speci fication allows to model how agents assign properties to objects based on their perception/perspective, rather than assigning salience to pre-determined features of these objects. These properties can totally, or partially, belong to the objects consistently with the basic intuition of fuzzy logic[7] .

From a coalgebric perspective, the role of above context-dependent param- eters, though which option-context pairs are built, can be played by allomor- phisms. They can operate as structure-altering maps that re-shape economic processes and games. In order to exemplify how allomorphisms can do that, in the next Section, we shall focus on two particular cases.

5. On Relational Allomorphisms

Perspective functions and membership functions help us to understand how morphisms can intervene in decision processes. The former are able to summa- rize di erent interpretative perspectives or positions, the latter give form to the idea that a property can only partially belong to an object. In what follows, by using these functions as examples, we shall build some relational allomorphisms for economic problems in which the order structure among objects matters. [21]

Take a nite, non-empty and ordered choice set (X, 4), and $k = 1, ..., l$ possible co-existing perspectives/properties on the choice process at hand. Let X be the Cartesian product between X_k with $k = 1, ..., l$.In the context K, a perspective function, $\Gamma : (x, K) \rightarrow X$, maps how di erent properties are assigned to choice option x X in context K. The bundle of properties indicates inherent features of option-context pairs that influence choice. [21]

See Bauman [16].The idea of fuzzy set was originally introduced by Zadeh. [22] For a primer in fuzzy logic see Nguyen et al. [23]

Let $C : X \rightarrow M(X) \subseteq X$ be a maximal-set choice function and, for each $x \in X$, ${x^*}_K{}^i = (x_1{}^i, ..., x_l{}^i)$ and ${x^*}_K{}^j = (x_1{}^j, ..., x_l{}^j)$ two different interpretations of relevant properties in context K, that are, two different outputs of Γ. If ${x^*}_K{}^i \equiv {x^*}_K{}^j$, then there must be an isomorphism f such that if $x \gtrsim y$, then $f(x) \gtrsim f(y)$ with $y \in X$, and $\Gamma(x, K) = \{{x^*}_K{}^i, {x^*}_K{}^j\}$. Thus, $M(X) = M(X^*)$ when properties $k = 1, ..., l$ hold.

On the contrary, if ${x^*}_K{}^i \neq {x^*}_K{}^j$ different interpretations of x can modify the order structure between alternatives. In this case, for instance, take an allomorphism η. Since X has finite length, we can define a rank operator as a function $rk: X \rightarrow I+$ such that, for $x, y \in X$, $rk(x) > rk(y)$ if $x \gtrsim y$, and $rk(x) = rk(y) + 1$ if y covers x. Then, an allomorphism yields

For at least one $x \in X$ or, alternatively, it makes true that if $x \succsim y$ for some $x, y \in X$, then $\eta(x) \lesssim \eta(y)$ with $x, y \in X$. Hence, it is allowed that if $x \succsim y$ then $\eta(x^{*}_{K}{}^{i}) \gtrsim \eta(y) \gtrsim \eta(x^{*}_{K}{}^{j})$ in context K. Consistently, it can also be that $M(X^{*}) \in X - M(X)$: the possibility of description-dependent, or perspective-dependent, choices is not ruled out.

Until now, we have implicitly supposed that a relevant property k belongs entirely to objects of choice. As mentioned above, a different way to approach the membership issue is through fuzzy logic. According to fuzzy sets theory, given a class of objects X and a property of these objects, say k, any $x \in X$ can partially, or totally, belong to the fuzzy set Δ_k, i.e., an element can partially, or totally, possess the property k.

A membership function of the kind:

$$\mu_{\Delta k}: X \rightarrow [0, 1]$$

with $\mu\Delta_k(x) = 0$ if x does not have the property k and $\mu\Delta_k(x) = 1$ if the property fully belongs to x, completely describes how partial membership is modeled.

Membership functions can be also used to build fuzzy orderings8 . Following Gonga et al. (2018), we can de ne a fuzzy order relation P on X as characterized by a membership function.

$$\mu^{P} : X \times X \rightarrow [0, 1] , \mu^{P} (x^{i}, x^{j}) = p_{ij}$$

verifying that

$$p_{ij} + p_{ji} = 1$$

and with $p_{ij} = 0$ if x^j is de nitely preferred to x^i, $p_{ij} = 1$ if the opposite holds, $p_{ij} = 0.5$ if x^j is indi erent to x^i, $p_{ij} \in [0, 0.5)$ if x^j is partially-preferred to xiand pij (0.5, 1] in the remaining case. Hence, with respect to a given property k in context K, we can rank alternatives by using the ordering gener- ated by $P\Delta_k$.

The idea of a fuzzy order relation was originally introduced by Bezdek et al. [24] and Nurmi. [25]

Recent advances in causal reasoning have given rise to a computation model that emulates the process by which humans generate, evaluate and distinguish counterfactual sentences. It is compatible with the "possible worlds" account according

The main advantage of fuzzy orderings lies in the possibility that the or- der between two options is reversed when reasons for partially-preferring one alternative over the other become counterfactuals. In logic, couterfactuals are conditional statements with a false antecedent [26], or conditionals interpreted as entailing that their antecedents are false. When referred to propositions, they have a puzzling element of inde niteness9 . With traditional order relations, the proposition x is de nitely preferred to y entails that y is de nitely dis-preferred to x or, alternatively, that not-having y is de nitely preferred to not-having x .

The last contraposition is not necessary true with a fuzzy order relation. In fuzzy logic, the statement xj is partially-preferred to xi , with, for instance, a crisp value of 0.4, can countefactually mean: there can be 4 cases on 10 in whichxj is not actually better that xi . In context K, if the last counterfactual holds, the attribute k of available choice alternatives can be reversely interpreted and this can change the resulting ordering between options10 . In such a case, an allomorphims of the kind:

$\eta : [0, 1] \rightarrow [0, 1]$

which yields that with $x^i P\Delta_k x^j$ and

$\mu^{P} (\eta(x^i) , \eta(x^j)) = p_{ji}$

it is also true that $\eta(x^j) \; P\Delta_k \, \eta(x^i)$.

Such a transformation can be interpret this transformation in terms of imaging (Lewis (1973)), i.e., a process of mass-shifting among possible worlds, provided that (i) worlds with equal histories should be considered equally similar and (ii) equally-similar worlds should receive mass in proportion to their prior status (Pearl (2000)).

In coalgebric terms, allomorphisms sketched above determine that:

$$(\pi > \eta) (s) (c) = C (\eta) (\pi (s) (c)) /= (\pi > f) (s) (c) = C (id + f) (\pi (s) (c)) \qquad (5)$$

or

$$(\eta > \pi) (s) (i) = \pi (s) (\eta (i)) /= \pi (s) (f (i)) \qquad (6)$$

In the first case, process outcomes are sensitive to di erent descriptions of outputs and nal states. In the second one, they are dependent on how inputs and starting conditions are intepreted.

6. A Final Remark

In this paper, we have discussed how to extend economic models for using the coalgebric language. A coalgebric set-up has internal consistency, formalism and direct code implementability, all features that make it very useful in the devel- opment of agent-based, or AI, models of economic behavior. Object-oriented languages are the main kind of language in these models which simulate data that trace the interactions of the programmed agents. The data is then used to statistically infer the properties of the composed systems11 . As we have seen, morphisms express relations between objects in categorical terms, and, hence, they can be easily injected in agent-based economic modeling.

Furthermore, morphisms can give account of agents' ability to re-construct and interpret properties by self-participating universals which sustain self-referential structures12 , i.e., interactions of the object and its encoding syntax and seman- tics. By creating rooms for these interactions, formal models focused on cate- gories and (allo)morphisms can contribute to the development, and increased usage, in economics of ontologies. The ontological embeddedness, or located- ness, of economic processes can imply that di erent spatio-temporal stimuli applied to the same process, and the same stimulus applied to di erent pro- cesses, produce di erent behavioral patterns, all having a general ontological status. As we have discussed, such a shift of perspective is needed for making economic models more relational and less substantialist.

to which there some constraints that are peculiar to exactly those transformations originate from actions. Lewis [27] formulation of counterfactuals indeed identifies such constraints: the transformation must be an imaging operator. See on the use of counterfactuals in computer science Pearl [28].

For instance Balke and Pearl [29] use dual networks, one representing the actual world, the other the counterfactual world to explore this possibility.

Counterfactuals are becoming a standard for explaining automated decisions, cfr. Wachter et al. [30] or Miller. [32]

Object orientation is common in programming languages; it has been formalized by coalgebraic semantics in theoretical computer science. See, for instance, Jacobs and Poll. [33] See Baas and Emmeche. [34]

Acknowledgement

Nill

Funding

No funding was received to carry out this study.

References

1. S. Awodey, Category Theory , Oxford University Press, New York, (2010).
2. N.A. Baas, C. Emmeche, On Emergence and Explanation , Intellec tica, 25 (1997) 67-83
3. A. Balke, J. Pearl, "Counterfactuals and Policy Analysis in Structural Models," in P. Besnard, S. Hanks, Uncertainty in Arti cial Intelligence 11 , Morgan Kaufmann, San Francisco, (1995).
4. Z. Bauman, Hermeneutics and Modern Social Theory , in David Held D., Thompson J.B. (eds) Social Theory of Modern Societies: Anthony Giddens and His Critics , Cambridge University Press, Cambridge, (1989).
5. J.C. Bezdek, B. Spillman, R. Spillman, A Fuzzy Relation Space for Group Decision Theory , Fuzzy Sets and Systems, 1 (1978) 255-268.
6. A. Blumensath, V. Winschel, A Coalgebraic Framework for Games in Economics , mimeo, (2013).
7. D. Borie, D. Jullien, Description-dependent Preferences, Journal of Economic Psychology, 81 (2020) 213-232.
8. R. Crespo, F. Tohme, The Future of Mathematics in Economics: A Philosophically Grounded Proposal , Foundations of Science, 22 (2017) 677-693.
9. F. Dietrich, C. List, Reason-based Choice and Context-Dependence: an explanatory framework , Economics and Philosophy, 32 (2016) 175-229
10. N. Elias, What Is Sociology? , Columbia University Press, New York, (1978).
11. D. Ellerman, A Theory of Adjoint Functions with some thoughts about their philosophical signi cance , What is Category Theory?, 3 (2006).
12. M. Emirbayer, Manifesto for a Relational Sociology , American Journal of Sociology, 103 (1997) 281-317.
13. A. Friedenberg, M. Meier, On the Relationship between Hierarchy and Type Morphisms , Economic Theory, 46 (2011) 377-399.
14. Z. Gonga, N. Zhangb, F. Chiclana, The Optimization Ordering Model for Intuitionistic Fuzzy Preference Relations with Utility Functions ,Knowledge-Based Systems, (2018).
15. J. Hedges, N. Ghani, A Compositional Approach to Economic Game Theory , Studia Logica, 82 (2006) 1-27.
16. D. Kahneman, A. Tversky, The Framing of Decisions and the Psychology of Choice , Science, 211 (1981) 453-58.
17. Y. Kamitake, Fundamental Concepts for Economic Systems The ory , Hitotsubashi Journal of Economics, 50 (2009) 181-192.
18. B. Kogelmann, What we choose, what we prefer , Synthese, 195 (2018) 3221-340.
19. B. Jacobs, E. Poll, Coalgebras and Monads in the Semantics of Java , Theoretical Computer Science, 291 (2002).
20. A.R. Lacey, A Dictionary of Philosophy , Routledge, London, (1986).
21. D. Lanzi, Embedded Choices , Theory and Decision, 68 (2010) 263-280.
22. D. Lewis, Counterfactuals , Basil Blackwell, Oxford, (1973).
23. S. Mac Lane, Categories for the Working Mathematician , Springer, Berlin, (1998).
24. J.P. Marquis, Category Theory ,in Zalta E. (ed.) The Stanford Encyclopedia of Philosophy, (2015).
25. G.C. Marshall, A. Freitas, Measuring Diagram Quality through Semiotic Morphisms , Semiotics, 239 (2021) 125-145.
26. T. Miller, Explanation in Arti cial Intelligence: Insights from the Social Sciences , Articial Intelligence, 267 (2019) 1-38.
27. H.T. Nguyen, C.L. Walker, A.E. Walker, A First Course in Fuzzy Logic , Taylor & Francis, New York, (2018).
28. H. Nurmi, Approaches to Collective Decision Making with Fuzzy Preference Relations , Fuzzy Sets and Systems, 6 (1981) 249-259.
29. S.E. Page, The Dierence: How the Power of Diversity Creates better Groups, Firms, Schools, and Societies , Princeton University Press,Princeton, (2007).
30. J. Pearl, Causality: Models, Reasoning, and Inference , Cambridge University Press, New York, (2000).
31. J. Pearl, The Algorithmization of Counterfactuals , Annals for Mathematics and Articial Intelligence, 61 (2011) 29-39.
32. M. Pint, Invariance under Type Morphisms: the Bayesian-Nash Equilibrium , mimeo, (2017).
33. D.A. Schmidt, Abstract Interpretation from a Denotational-semantics Perspective , Electronic Notes in Theoretical Computer Science, 249 (2009) 19-37.
34. P.A. Streufert, The Category of Node-and-Choice Extensive-form Games , mimeo, (2007).