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A stochastic approach to forecast project duration

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Abstract

For executives who track projects on a complete / failed basis, at each phase of the project, the project completion date may be adjusted using only stochastic calculations in closed form. The proposed method to predict the final project duration may be interpreted intuitively. The comparative analysis of the effectiveness of the proposed method shows a good computational accuracy of predictions in many cases surpassing the accuracy of existing methods.

Keywords: Forecasting, Earned Value Management (EVM), Earned Schedule (ES), Planned duration, Budget at completion, Performance measurement baseline, Compound Poisson process, stopping time.

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1.INTRODUCTION

First introduced in 2003, Earned Schedule method (ES) addresses some of the shortcomings and expands on the benefits of Earned Value Management method (EVM). Compared to the EVM method, the ES method enables easier analysis of the schedule and predicting remaining project durations.

To be aware of the project completion date is important in order to take management decisions correctly.However, existing forecasting methods, known to the authors, are of empirical nature. Forecasting formulas are based neither on any scientific approach nor are probabilistic, which seems unconvincing for forecasting.

In many cases the process of project execution may be regarded as a piecewise compound Poisson process: there is a series of non-decreasing compound Poisson processes $X_k(t)$, and a series of increasing levels PV_k so that when at the stopping time s_k the process $X_k(t)$ reaches the level PV_k, $X_k(t)$ stops and the process $X_{k+1}(t)$ starts from the level PV_k. In the given situation we deduce forecasting formulae and indicate possible solutions for the project manager.

The proposed new method to predict the final project duration is interpreted intuitively.

2.Setting of the problem

Following the approach of the ES method, we consider two curves - the Performance Measurement Baseline (PMB) curve and the Earned Value curve (EV).

Denote the values of the planned non-decreasing levels by PV_k, k = 0,…, N, corresponding to the given increasing values of the time T_k, k = 0,…, N. We assume T_0= 0, PV_0 = 0. The final values are defined as T_N = PD, PV_N = BAC, where PD is the planned duration of the project (Planned Duration) and BAC is the Budget At Completion.

These settings are shown in Figure 1.

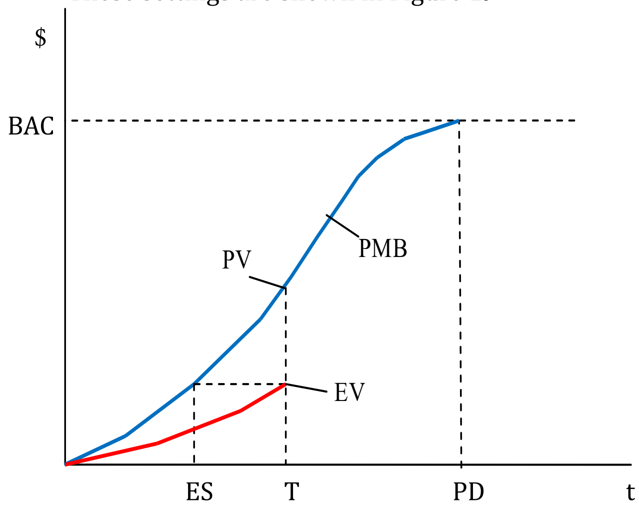

Figure 1 - Planned and earned values (qualitatively)

For many projects we may assume that within each phase of the project its actual realization is described by Lévy process:[1,2] the independence and stationarity of increments may be reached by subdivision into smaller phases where we can neglect the nature of work; by separating "calendar effects" into a separate group, the rest of the project remains stochastically continuous.

For executives who track schedules on a complete / failed basis, an actual path is always a piecewise constant. However, Lévy processes with piecewise constant paths are compound Poisson processes.[1] Also we assume that our projects have non- decreasing paths.

In this paper we concentrate on projects presented with piecewise compound Poisson processes with non-decreasing trajectories. Mathematically, the project $X(t)$ is set by the sequence of the compound Poisson processes with non-decreasing paths $X_k(t)$, $k = 1, \dots, N$, the increasing sequence of the numbers $0=PV_0 < PV_1 < \dots < PV_N=BAC$, and the time MaxT by which we consider all the processes of interest are finished. This time is introduced just to guarantee that all stopping times are finite.

Next, we introduce stopping times $s_k=\inf\{t \mid X_k(t)=PV_k\} \wedge MaxT$.

The realisation of the project starts as the process $X_1(t)$ at (T_0, PV_0), $T_0=0$, $PV_0=0$ until the stopping time s_1 hits. At s_1 the process $X_1(t)$ stops and the process $X_2(t)$ starts at $(s_1, X_1(s_1))$, so $X_2(s_1) = X_1(s_1)$, and so on. The project finishes as $X_N(t)$ at the stopping time s_N. Hence the graph of the expectation $\mathbb{E}[X(t)]$ corresponds to the line PV in the Figure 1 and it is named the schedule of the project $X(t)$. The expected values of the stopping times s_k, $T_k = \mathbb{E}[s_k \mid s_{k-1} = T_{k-1}]$, separate different phases of the schedule, while the stopping times s_k separate different phases of the project and the process within each phase is a compound Poisson process with non-decreasing trajectories. Notice that such processes may be shifted / expanded to any time intervals. Also, notice that in this case, the graph of the expected schedule PV is piecewise linear: points (T_j, PV_j) and (T_{j+1}, PV_{j+1}) define linear intervals.

The graph of expected values is piecewise linear, and on each linear interval

$\mathbb{E}[X_k(t) \mid s_{k-1} = T_{k-1}] = C_{k-1} + \mu_k t$,

with $\mathbb{E}[X_k(T_{k-1}) \mid s_{k-1} = T_{k-1}] = PV_{k-1} = C_{k-1} + \mu_k T_{k-1}$, and $\mathbb{E}[X_k(T_k) \mid s_{k-1} = T_{k-1}] = PV_k = C_{k-1} + \mu_k T_k$, $t \in [T_{k-1}, T_k]$, $k = 1, \dots, N$. μ_k are the slopes of the line connecting the points (T_{k-1}, PV_{k-1}) and (T_k, PV_k). Notice that compensating Lévy processes $X_k(t) - \mu_k t$, $t \in [T_{k-1}, T_k]$, $k = 1, \dots, N$, are martingales within their intervals: $\mathbb{E}[X_k(t) - \mu_k t \mid s_{k-1} = T_{k-1}] = PV_{k-1} - \mu_k T_{k-1}$. For convenience, we put $\mu_0 = 0$, $s_0 = 0$, then $\mathbb{E}[X_k(s_k) - \mu_k s_k \mid s_{k-1} = T_{k-1}] = PV_{k-1} - \mu_k T_{k-1}$. Therefore, in conditional expectations we have

$$\mathbb{E}[s_k \mid s_{k-1} = T_{k-1}] = T_{k-1} + (PV_k - PV_{k-1}) / \mu_k. \quad (1)$$

3.Forecasting in the course of project execution

Let's suppose we start implementing the planned activities phase by phase. According to the schedule, at the moment T_b, we should complete b phases of the project reaching the value PV_b, in reality, however, we reach the level $EV_b = X(T_b)$. Our goal is to estimate the future values of stopping times

$\mathbb{E}[s_k \mid EV_b = X(T_b)]$.

In case when $EV_b = PV_b$, the project realization coincides with the planned schedule and our expectation coincides with the planned schedule.

In case of delay, $EV_b = X(T_b) < PV_b$, first we should define the current phase of the project at the time T_b using inequalities

$PV_{j-1} \le X(T_b) < PV_j$, $j \le b$

While predicting the future we start from the compound Poisson process $X_j(t)$. The equation (1) shows that the expectations of the differences of stopping times are proportional to the levels increments. Similar considerations result in the equation

$\mathbb{E}[s_j - T_b \mid EV_b = X_j(T_b)] = (PV_j - EV_b)/\mu_j$.

The process $X_j(t)$ will perform until the stopping time s_j and the expectation

$\mathbb{E}[s_j \mid EV_b = X_j(T_b)] = T_b + (PV_j - EV_b)/ \mu_j$ hitting at the level PV_j.

Next, the process $X_{j+1}(t)$ will start from the initial level PV_j. The process $X_{j+1}(t)$, having the slope μ_{j+1}, will run until the stopping time s_{j+1}. Expectation of the time duration remains the same

$\mathbb{E}[s_{j+1} - s_j \mid EV_b = X_j(T_b)] = T_{j+1} - T_j$.

Proceeding the same way we see that time increments remain unchanged,

$\mathbb{E}[s_{k+1} - s_k \mid EV_b = X_b(T_b)] = T_{k+1} - T_k$, for all $k \ge j$.

Finally, for the case of delay, our formulas are

$\mathbb{E}[s_j - T_b \mid EV_b = X_j(T_b)] = (PV_j - EV_b)/ \mu_j$,

$\mathbb{E}[s_{j+1} - T_b \mid EV_b = X_j(T_b)] = T_{j+1} - T_j + (PV_j - EV_b)/ \mu_j$,

$$\mathbb{E}[s_k - T_b \mid EV_b = X_j(T_b)] = T_k - T_j + (PV_j - EV_b)/ \mu_j, \ k \ge j. \quad (2)$$

Total time lag is $dT_b = T_b - T_j + (PV_j - EV_b)/ \mu_j$. These considerations allow a simple geometric interpretation of the adjusted forecast: to get the adjusted schedule as seen at the time T_b the original piecewise linear graph should be shifted to the right by dT_b. The expectation of the project duration is increased by the same amount dT_b:

$\mathbf{RD = PD + dT_b}$.

A graphical interpretation is shown in Figure 2.

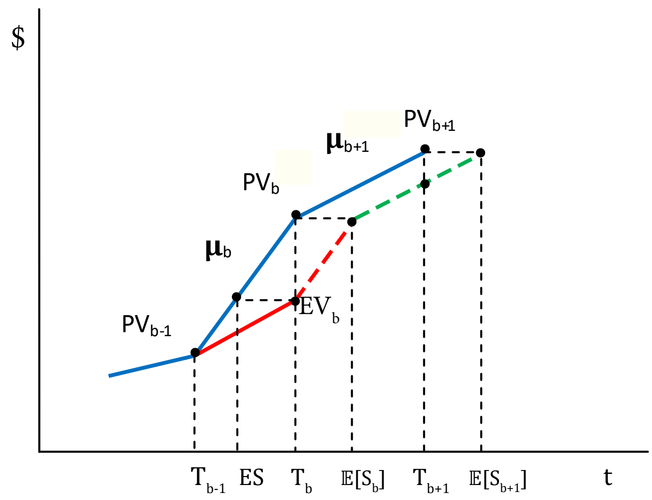

Figure 2 - The forecast for the lagging data (j=b)

Figure 2 shows ES time, the earned schedule time that refers to the earned schedule method. The time lag is

$\mathbf{dT_b = T_b - ES}$.

Thus, the forecast for a future should be adjusted: the volumes of the planned work remain the same, but the completion times are lagged by dT_b. As a rule, such a correction is carried out if the deviation from the previously developed plan does not exceed 10% in terms of time or volume of work performed.

Now let's consider the case when at the time t the works are ahead of the schedule, $EV_b = X(T_b) > PV_b$.

Again, the current phase j is defined by means of inequalities $PV_{j-1} \le X(T_b) < PV_j$,

In this case, at the time T_b, $j > b$.

Again $\mathbb{E}[s_j - T_b \mid EV_b = X_j(T_b)] = (PV_j - EV_b)/ \mu_j$.

All prior formulas remain valid:

$\mathbb{E}[s_{k+1} - s_k \mid EV_b = X_b(T_b)] = T_{k+1} - T_k$, for all $k \ge j$, and

$\mathbb{E}[s_k - T_b \mid EV_b = X_j(T_b)] = T_k - T_j + (PV_j - EV_b)/ \mu_j$, $k \ge j$.

As in case of time lag, the time shift equals to $dT_b = T_b - T_j + (PV_j - EV_b)/ \mu_j$. But as $T_b < T_j$, when ahead of the schedule, $dT_b < 0$. Now, to adjust the forecast given at the time T_b, we should shift the original piecewise linear graph to the left by $-dT_b$. The expected project duration decreases by the same value: **RD = PD - dT$_b$.**

A graphical interpretation of the described calculations is shown in the demo Figure 3.

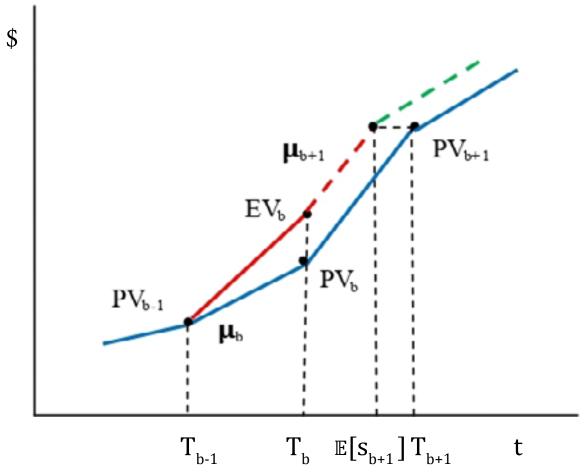

Figure 3 - Forecasting when working ahead of the schedule (j=b+1)

The proposed forecasting method for final project duration will be further referred to as parallel shift (PS) method.

4.Comparative analysis of the effectivity of the PS method

In order to test the forecasting capabilities of the PS method, five different execution scenarios were simulated. The scenarios differed from each other in the duration of the completion, success of their completion (on time, prior or posterior ending, as well as the progress of the project itself. The following goal was set at that stage:

To show the possibility of predicting the final duration of the project, assessing the complexity of calculations and comparing this method with the well-known ES method [3].

The data from all five projects were used to generate the forecast using two methods: the first method one was ES, and the second one was the PS method. The calculations by the ES method are described in the article of W.Lipke.[3]

For calculations by the PS method, the above formulae were used. These forecasts were then analyzed and compared with each other. The forecast accuracy was calculated in the process of analyzing. The accuracy of the forecast was determined using the standard deviation of the predicted value of the final duration of the project, calculated by the appropriate method, from actual duration of the project. [4]:

$$\sigma_m = [\Sigma\ (IEAC_m(i) - FD)^2/(n-1)]^{0,5},$$

where

σm is the standard deviation for the prediction method m; **IEACm(i)** is the predicted value for method m at point i; FD is the actual final duration of the project;

n is the number of status points;

Σ - summation over a set of state points.

As a result of the simulation, in four cases out of five, the accuracy of estimating the final duration of the project by the parallel shift method is higher than by the earned schedule method (see the Table below).

Xj(t)

Table Standard deviations of forecasting the final duration of projects

Project progress Scenarios	σ for ES method	σ for PS method
№1	1,79	0,69
№2	1,48	1,05
№3	1,24	1,29
№4	2,32	0,87
№5	1,07	0,59
Sum of σi	7,9	4,49

Figure 4 shows the diagram for one of the project execution scenarios (scenario 4). And only in one case, the accuracy of the parallel shift method was slightly worse than that of the earned schedule method (this case is highlighted in the Table in yellow). Moreover, the integral value of the standard error criterion for all five scenarios for the parallel shift method is 1.76 times lower than for the earned schedule method (Table and Figure 5).

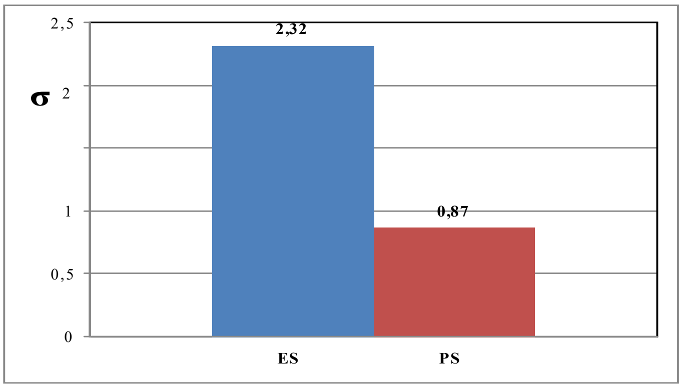

Figure 4 - The standard deviation of the predicted value of the final duration of the project, calculated by the earned schedule method (ES) and the parallel shift method (PS) for scenario #4

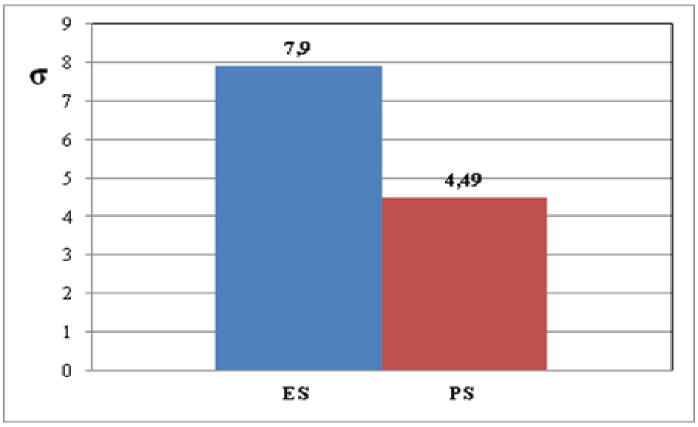

Figure 5 - Total standard deviation of the predicted value of the final duration of the project, calculated by the earned schedule method (ES) and the parallel shift method (PS) for scenarios ##1-5.

If we continue to compare the proposed PS method with the ES method, then the proposed PS method has an additional opportunity, which consists in determining the magnitude of the shift of control points (milestones) in the project schedule, which, in fact, is determined by the calculation method itself.

There is one more fundamental difference between ES and PS methods. When predicting the final duration of the project using the ES method, the ratio of the time of the earned schedule to the current time is actually found, and in accordance with this ratio, the final duration of the project increases (or decreases). With the same ratio, it is obvious that the longer this duration, the more the forecast value differs from the planned duration of the project and, accordingly, the forecast error will be higher. In contrast to the ES method, with the PS method, the forecast is determined by the accumulated difference between scheduled and executed duration of the project. And, in this case, the forecast, and accordingly, the forecast error will not depend on the remaining duration of the project.

5.Conclusion

This study examined predicting the final duration of a project.The authors showed that in many cases each phase of the project may be represented by the non- decreasing compound Poisson process. Under these assumptions the planned schedule is represented by a piecewise linear non-decreasing curve, which quite accurately corresponds to the practice of project execution. Under these assumptions we deduce how to forecast the project duration and the project schedule during the project execution. The method which uses the described methodology, the authors called the parallel shift (PS) method.

In order to compare this method to the existing ones, several scenarios were analysed. For each scenario, the forecast of the final duration of the project at control points was calculated by two methods - by the new PS method, and by one out of the best previously known methods, the ES method. In one out of five trials the error of the ES method was slightly smaller than the error of the PS method. In all other trials the error of the PS method was significantly lower. Despite the fact that the given data were obtained from a small number of the realizations, it can be argued that the proposed method has a good computational accuracy, comparable to the accuracy of predictions obtained by the ES method, and in many cases even surpassing it.Using the PS method, you can determine the time offset of the milestones of the project schedule.Unlike the ES method, in case of the PS method, the forecast error does not depend on the duration of the project. In addition, the forecast obtained using the parallel shift method is probabilistic in nature and has a very simple and understandable interpretation.

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References

1. R. Cont, P. Tankov, Financial Modeling With Jump Processes, Chapman & Hall/CRC, (2004).
2. И.И. Гихман, А.В. Скороход, В.М. Введение теорию случайных процессов,http://booksshare.net/books/physics/gihman-ii/1977/files/vvedenievteoriusluchaynihprocessov1977.pdf, (1997)
3. W. Lipke, Earned Schedule Contribution to Project Management. PM World Journal, Vol. I(2) 2012.
4. W. Lipke, Project Duration Forecasting, A Comparison of Earned Value Management Methods to Earned Schedule, The Measurable News , 2 (2009) 24–31.