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Bayesian estimation of auc of a constant shape bi- weibull failure time distribution

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ABSTRACT
A Receiver Operating Characteristic (ROC) curve provides quick access to the quality of classification in many medical diagnoses. The Weibull distribution has been observed as one of the most useful distributions, for modeling and analyzing lifetime data in Engineering, Biology, Survival and other fields. Studies have been done vigorously in the literature to determine the best method in estimating its parameters. In this paper, we examine the performance of Bayesian Estimator using Jeffreys‘ Prior Information and Extension of Jeffreys‘ Prior Information with three Loss functions, namely, the Linear Exponential Loss, General Entropy Loss, and Square Error Loss for estimating the AUC values for Constant Shape Bi-Weibull failure time distribution. Theoretical results are validated by simulation studies. Simulations indicated that estimate of AUC values were good even for relatively small sample sizes (n=25). When AUC≤0.6, which indicated a marked overlap between the outcomes in diseased and non-diseased populations. An illustrative example is also provided to explain the concepts.

Key words: AUC, Biomarker, Constant Shape Bi-Weibull ROC model, Bayesian Method, Jeffreys‘ Prior Information, Extension of Jeffreys‘ Prior Information.

1. INTRODUCTION

Receiver operating characteristic (ROC) curves are widely used for the evaluation of continuous or ordinal diagnostic tests and biomarkers [4, 6, 20]. The ROC Curve is embedded by the two intrinsic measures Sensitivity (Sn) or True Positive Rate (TPR) and 1-Specificity (Sp) or False Positive Rate (FPR) along with its accuracy measure AUC. The corresponding AUC provides a global summary statistic indicating the overall discriminatory ability of a test independent of any cut-off point, and may be used to compare the performance of different tests for detection of the condition of interest.

The AUC is defined as the total Area under the ROC curve (AUC) represented by ―sensitivity‖ versus ―1-specificity‖ corresponding to all possible cut-off points. The AUC can be interpreted as the average sensitivity, across all possible False-Positive Fractions [26]. An alternative interpretation is the proportion of the time that test scores for individuals with the condition of interest will exceed (or be less than) those of individuals without the condition.

The term ROC analysis was coined during II world war to analyze the radar signals [19]. The application of ROC Curve technique was promoted in diversified fields such as experimental psychology [7], industrial quality control [3] and military monitoring [24]. [7] Was first to use the Gaussian model for estimating the ROC Curve. The importance of ROC Curve in

medicine was due to [16] analyze the radiographic images. [9] Explained the importance and robustness of Binormal ROC Curve. Recently [21] was explain the Bayesian estimation of the receiver operating characteristic curve for a diagnostic test with a limit of detection in the absence of a gold standard.

We also developed Functional Relationship between Brier Score and Area Under the Constant Shape Bi-Weibull ROC Curve [15], Confidence Intervals Estimation for ROC Curve, AUC and Brier Score under the Constant Shape Bi-Weibull Distribution [12], Asymmetric and Symmetric Properties of Constant Shape Bi-Weibull ROC Curve Described by Kullback-Leibler Divergences [13], and Bayesian Estimation of Parameters under the Constant Shape Bi-Weibull Distribution Using Extension of Jeffreys‘ Prior Information with Three Loss Functions [14].

The main purpose of this paper is to compare the Estimation of the AUC of the Constant Shape Bi-Weibull distribution with its Bayesian counterpart using Extension of Jeffreys‘ Prior Information and Jeffreys‘ Prior Information obtained from Lindley‘s approximation procedure based on three Loss Functions.

In this paper, the Bayesian Estimation of AUC under the Constant Shape Bi-Weibull Distribution is studied by Using Extension of Jeffreys‘ Prior Information and Jeffreys‘ Prior Information with Three Loss Functions. This paper is organized as follows: In Section 2, estimation of AUC under Jeffreys‘ Prior Information and Extension of Jeffreys‘ Prior Information with Three Loss functions is discussed. Section 3, provides simulation study for proposed theory. Section 4 deals with validation of the proposed theory based on real time data. Conclusions are given in Section 5.

2. A CONSTANT SHAPE BI-WEIBULL ROC MODEL AND ITS AUC

In medical science, a diagnostic test result called a biomarker [10, 1] is an indicator for disease status of patients. The accuracy of a medical diagnostic test is typically evaluated by sensitivity and specificity. Receiver Operating Characteristic (ROC) curve is a graphical representation of the relationship between sensitivity and specificity. Hence the main issue in assessing the accuracy of a diagnostic test is to estimate the ROC curve.

Suppose that there are two groups of study subjects: diseased and nondiseased. Let S be a continuous biomarker. Assume that ROC analysis based on the True Positive Probability (TPP), | , and False Positive Probability (FPP), | , in fundamental detection problems with only two events and two responses [25,7,2].

According to Signal Detection Theory(SDT), we assume that there are two probability distributions of the random variables X and *Y*, one associated with the signal event s and other with the non signal event n[11]; these probability (or density) distributions of a given observation *x and y* are conditional upon the occurrence of *s* and *n* [2].

In the medical context, the signal event corresponds to the diseased group, and the nonsignal event to the nondiseased group [8]. If the cutoff value is *c*, corresponding to a particular likelihood ratio, the TPP and FPP are given by the following expressions [2]:

Let x, *y* be the test scores observed from two populations with (diseased individuals) and without (nondiseased individuals) condition respectively which follow Constant Shape Bi-Weibull distributions. The density functions of Constant Shape Bi-Weibull distributions are as follows,

and

The probabilistic definitions of the measures of ROC Curve are as follows:

In this context, the (1-Specificity) and Sensitivity can be defined using equations (1) and (2) and are given in equations (3) and (4) respectively,

and

The ROC Curve is defined as a function of (1-Specificity) with scale parameters of distributions and is given as,

where [ ] is the threshold and

The accuracy of a diagnostic test can be explained using the Area under the Curve (AUC) of an ROC Curve.AUC describes the ability of the test to discriminate between diseased and no diseased populations.

A natural measure of the performance of the classifier producing the curve is AUC. This will range from 0.5 for a random classifier to 1 for a perfect classifier. The AUC is defined as,

$$\int$$

The closed form of AUC is as follows

$$\text{—}$$

2.1 Bayesian Estimation of AUC under Constant Shape Bi-Weibull Distribution

For analysing Failure time data Bayesian estimation approach has received a lot of attention. It makes use of once prior knowledge about the parameters and also takes into consideration the data available. If once prior knowledge about the parameter is available, it is suitable to make use of an informative prior but in a situation where one does not have any prior knowledge about the parameter and cannot obtain vital information from experts in this regard, then a non-informative prior will be a suitable alternative to use, Guure et al. [5].

Let ...... be a random sample of size m from and be a random sample of size n from .The likelihood function of the selected sample is given by

$$| \quad \prod \quad | \quad \prod \quad |$$

where

$$\prod - \quad {}^{*}\!+\!\!- \prod \quad - \quad {}^{*}\!\!-\!\!+$$

The log-likelihood function is

$$[\sum \quad \sum] \quad -\sum$$

$$-\sum$$

The Bayes estimators of the parameters are considered with different loss functions which are given below:

$$(\hat{\ } \quad) \quad ((\hat{\ } \quad)) \quad (\hat{\ } \quad)$$

$\hat{(} \quad) \quad \hat{()}- \quad \hat{(-)}$

and

$\hat{()} \; \hat{( )}$

a) Jeffreys Prior information

Consider a likelihood function L(θ), with its Fisher Information . The

Fisher Information measures the sensitivity of an estimator. Jeffreys (1961) suggested that

[ ] be considered as a prior for the likelihood function L(θ).

The Jeffreys prior is justified on the grounds of its invariance under parameterization according to Sinha [22]. Under the two-parameter Weibull distribution the non-informative (vague) prior according to Sinha and Sloan [23] is given as

$[—]$

Let the likelihood equation which is | be the same as (6). The joint posterior of the parameters ***σ*** and **β** is given by

$| \quad |$

The marginal distribution function is the double integral of equation (11). Therefore, the posterior probability density function of σ and β given the data ($t_1, t_2, \ldots, t_n$) is obtained by dividing the joint posterior density function over the marginal distribution function as

$$| \quad \frac{|}{\iint \quad |}$$

b) Extension of Jeffreys Prior information

We propose a Extension of Jeffreys Prior information such that,

$[-]$

This is a Extension of Jeffreys Prior information, when a = 1, we have the standard Jeffreys Prior information and undefined when a = 0. Since our knowledge on the parameters is limited as a result of which a Jeffreys Prior information approach is employed on both parameters, it is important that one ensures the prior does not significantly influence the final result. If our limited or lack of knowledge influences the results, one may end-up giving wrong interpretation which could affect whatever it is we seek to address. It is as a result of this that the Extension of Jeffreys Prior information is considered.

We have,

$[—]$

The likelihood function from equation (6) is

$$| \quad \prod - \quad ^{*}\_+$$

According to Bayes theorem, the joint posterior distribution of the parameters $\boldsymbol{\sigma}$ and $\boldsymbol{\beta}$ is

$$| \quad |$$

where $| \quad \text{———} \prod \quad ^{*}_{-}{}^{+} \quad —$

and the marginal distribution is $\iint \text{———} \prod \quad ^{*}_{-}{}^{+} \quad —$

where k is the normalizing constant that makes a proper pdf. The posterior density function is obtained by using equation (12).

Bayesian Estimation approach has received a lot of attention in recent times for analyzing Failure Time data, which has mostly been proposed as an alternative to that of the traditional methods.

Remark 2.1

Here we consider two Asymmetric Loss Functions namely Linear Exponential (LINEX) Loss Function and General Entropy Loss Function. Also the Symmetric Loss Function namely Squared Error Loss Function considered in order to estimate AUC values.

2.2.1 Linear Exponential Loss Function (LINEX)

The LINEX Loss Function is under the assumption that the minimal loss occurs at $\hat{}$ and is expressed as

$$(\hat{} \quad) \qquad ((\hat{} \quad)) \quad (\hat{} \quad)$$

where $\hat{}$ is an estimation of **θ** and .... The sign and magnitude of the shape parameter ‗**a**‘ represents the direction and degree of symmetry, respectively. There is overestimation if **a** > 0 and underestimation if **a** < 0 but when , the LINEX Loss Function is approximately the Squared Error Loss Function. The posterior expectation of the LINEX Loss Function, according to [18], is

$$(\hat{} \quad) \qquad (\hat{}) \qquad\qquad (\hat{} \qquad)$$

The Bayes Estimator of **θ**, represented by $\hat{}$ under LINEX Loss Function, is the value of $\hat{}$ which minimizes equation (13) and is given as

$$\hat{} \qquad -$$

provided exists and finite. The Bayes Estimator $\hat{}$ of a function is given as

$$\frac{\iint \quad [\quad]}{\iint}$$

From (14), it can be observed that ratio of integrals which cannot be solved analytically and for that we employ Lindley‘s approximation procedure to estimate the parameters. Lindley considered an approximation for the ratio of integrals for evaluating the posterior expectation of an arbitrary function ˆ as

$$[\,|\,] \frac{\int [\,]}{\int [\,]}$$

According to [22], Lindley‘s expansion can be approximated asymptotically by

where *L* is the log-likelihood function in equation (7),

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( ) Σ ( ) Σ ( )

Σ

Σ

Σ

Σ

For Extension of Jeffreys Prior information

For Jeffreys Prior information

Bayesian Estimation of AUC using LINEX Loss Function is given as

2.2.2 General Entropy Loss Function

Another useful Asymmetric Loss Function is the General Entropy (GE) Loss which is a generalization of the Entropy Loss and is given as

The Bayes Estimator of **θ** under the General Entropy Loss is

provided exists and finite.

The Bayes Estimator for this Loss Function is

Applying the same Lindley approach here as in (15) with u_{100}, u_{200}, u_{010}, u_{020} and u_{001}, u_{002} are the first and second derivatives for and $\boldsymbol{\beta}$, respectively, and are given as

[ ] — [ ]

—

Bayesian Estimation of AUC using the General Entropy Loss is given as

^ ——— ^ ^ ^ ——

2.2.3 Symmetric Loss Function

The Squared Error Loss is given by

$\hat{(}\quad)\ \hat{(}\quad)$.

This Loss Function is symmetric in nature, that is, it gives equal weightage to both over and under estimation.

In real life, we encounter many situations where overestimation may be more serious than underestimation or vice versa.

The Bayes Estimator ^ of a function of the unknown parameters under Square Error Loss Function (SELF) is the posterior mean, where

$$\hat{}\quad \{\ [\quad]|\ \} \quad \frac{\iint [\ ]}{\iint}$$

Applying the same Lindley approach here as in (15) where u100, u200, u010, u020 and *u001*, *u002* are the first and second derivatives for and ***β***, respectively, and are given as

—

—

—

Bayesian Estimation of AUC using The Squared Error Loss is given as

^ ^ ^ ^

3. SIMULATION STUDY

Simulation studies are conducted with different combinations of scale and shape parameters of both diseased and non-diseased populations. At every parameter combination and sample size, the AUC is obtained. The main purpose of conducting simulations is to show how the AUC of ROC curve possesses different values as the scale and shape parameters of the normal and abnormal distributions change.

The AUC has been computed through different methods via Bayesian Estimation using Extension of Jeffreys' Prior Information and Jeffreys' Prior Information obtained from Lindley's approximation procedure with three Loss Functions.

In our Simulation study, we chose a sample size of ***n***= 25, 50, and 100 to represent small, medium, and large dataset. The assumed scale and shape parameters of both populations are . The values of Jeffreys' Extension are
and 1.4. The values for the Loss parameters **(a, k)** are ***a***=***k***=±0.6 and ±1.6.

In Table 1 we present the estimated values for AUC for Bayesian Estimation using extension of Jeffrey's prior information with the three loss functions.

Table 1: Bayesian Estimated values for AUC using extension of Jeffrey's prior information

(m, n)			*β*	*c*	^	^	^	^	^	^	^	^	^
						a=k=0.6		**a=k=-0.6**		**a=k=1.6**		**a=k=-1.6**	
(25,25)	1.5	0.5	0.8	0.4	0.7500	0.3399	0.3416	0.6497	0.6590	0.1280	0.1487	0.8258	0.8533
	1.5	0.5	1.2	0.4	0.7440	0.3536	0.3431	0.6438	0.6553	0.1631	0.1481	0.8253	0.8454
	1.5	0.5	0.8	1.4	0.7725	0.3307	0.3261	0.6603	0.6602	0.1179	0.1283	0.8450	0.8764
	1.5	0.5	1.2	1.4	0.7769	0.3266	0.3222	0.6632	0.6788	0.1100	0.1232	0.8487	0.8802
(50,50)	1.5	0.5	0.8	0.4	0.7534	0.3462	0.3400	0.6486	0.6611	0.1476	0.1472	0.8303	0.8573
	1.5	0.5	1.2	0.4	0.7464	0.3518	0.3426	0.6451	0.6566	0.1594	0.1487	0.8271	0.8486
	1.5	0.5	0.8	1.4	0.7648	0.3364	0.3322	0.6567	0.6694	0.1300	0.1368	0.8418	0.8693
	1.5	0.5	1.2	1.4	0.7600	0.3465	0.3328	0.6510	0.6666	0.1523	0.1348	0.8377	0.8629
(100,100)	1.5	0.5	0.8	0.4	0.7504	0.3519	0.3409	0.6462	0.6593	0.1615	0.1473	0.8305	0.8535
	1.5	0.5	1.2	0.4	0.7484	0.3528	0.3416	0.6455	0.6580	0.1632	0.1478	0.8296	0.8510
	1.5	0.5	0.8	1.4	0.7545	0.3501	0.3379	0.6483	0.6622	0.1590	0.1429	0.8343	0.8578
	1.5	0.5	1.2	1.4	0.7563	0.3486	0.3363	0.6494	0.6636	0.1561	0.1404	0.8357	0.8595

From Table 1 it is observed that Bayes estimator under LINEX and General Entropy Loss functions tend to underestimate the AUC values when loss parameters value are (0.6,1.6).

Bayes estimation with General Entropy loss function provides the highest AUC values when the loss parameter is -1.6, according to the extension of Jeffreys prior value is 0.4 or 1.4. So that the Bayes estimators of AUC under General Entropy loss function is best estimation method for Constant Shape Bi-Weibull Distribution.

Table 2: Bayesian Estimated values for AUC using Jeffrey's prior information

(m, n)			***β***	^	^	^	^	^	^	^	^	^
					a=k=0.6		**a=k=-0.6**		**a=k=1.6**		**a=k=-1.6**	
(25,25)	1.5	0.5	0.8	0.7521	0.3384	0.3400	0.6511	0.6605	0.1255	0.1464	0.8282	0.8554
	1.5	0.5	1.2	0.7452	0.3530	0.3421	0.6444	0.6562	0.1619	0.1468	0.8264	0.8467
	1.5	0.5	0.8	0.7543	0.3411	0.3394	0.6502	0.6618	0.1338	0.1465	0.8286	0.8582
	1.5	0.5	1.2	0.7531	0.3397	0.3397	0.6507	0.6611	0.1293	0.1462	0.8283	0.8566
(50,50)	1.5	0.5	0.8	0.7541	0.3456	0.3394	0.6492	0.6617	0.1466	0.1464	0.8313	0.8581
	1.5	0.5	1.2	0.7473	0.3512	0.3419	0.6457	0.6573	0.1585	0.1477	0.8281	0.8496
	1.5	0.5	0.8	0.7567	0.3421	0.3383	0.6513	0.6634	0.1390	0.1455	0.8332	0.8611
	1.5	0.5	1.2	0.7475	0.3523	0.3417	0.6453	0.6575	0.1613	0.1473	0.8284	0.8497
(100,100)	1.5	0.5	0.8	0.7507	0.3516	0.3406	0.6465	0.6595	0.1611	0.1470	0.8310	0.8539
	1.5	0.5	1.2	0.7489	0.3525	0.3413	0.6458	0.6583	0.1627	0.1473	0.8301	0.8515
	1.5	0.5	0.8	0.7505	0.3522	0.3407	0.6462	0.6593	0.1624	0.1470	0.8309	0.8536
	1.5	0.5	1.2	0.7498	0.3518	0.3409	0.6463	0.6590	0.1612	0.1470	0.8306	0.8527

From Table 2 it is observed that Bayes estimator under LINEX and General Entropy Loss functions tend to underestimate the AUC values when loss parameters value are (0.6,1.6).

Bayes estimation with General Entropy loss function provides the highest AUC values when the loss parameter is -1.6. So that the Bayes estimators of AUC under General Entropy loss function is best estimation method for Constant Shape Bi-Weibull Distribution.

4. ILLUSTRATION

The real data set namely Coronary Heart Disease [CHDAGE] Data extracted from [27].Data consists of 100 Observations, 3 variables. For this data we have to find estimate AUC values. This data consists of patients who are Diseased and who are Non-diseased. We have to know the patients with Diseased and patients without Diseased and age is most influential variable for diagnose. Table 2 depicts the Estimated AUC values using CHDAGE Data.

Table 3: Estimated AUC values using CHDAGE Data

	=1.5, =0.5, c=0.4		=1.5, =0.5, c=1.4	
(m, n)=(43,57)	*β=0.8*	*β=1.2*	*β=0.8*	*β=1.2*
MLE	0.5541	0.5792	0.5541	0.5792
EBS	0.7500	0.7500	0.7502	0.7500
JBS	0.7501	0.7500	0.7501	0.7500
EBL(a=k=0.6)	0.3542	0.3543	0.3540	0.3542
EBL(a=k=-0.6)	0.6456	0.6456	0.6457	0.6456
EBL(a=k=1.6)	0.1675	0.1679	0.1673	0.1678
EBL(a=k=-1.6)	0.8318	0.8319	0.8320	0.8320
EBG(a=k=0.6)	0.3409	0.3409	0.3408	0.3409
EBG(a=k=-0.6)	0.6590	0.6590	0.6592	0.6591
EBG(a=k=1.6)	0.1471	0.1470	0.1469	0.1470
EBG(a=k=-1.6)	0.8530	0.8529	0.8532	0.8529
JBL(a=k=0.6)	0.3542	0.3543	0.3542	0.3543
JBL(a=k=-0.6)	0.6456	0.6456	0.6456	0.6456
JBL(a=k=1.6)	0.1675	0.1678	0.1675	0.1678
JBL(a=k=-1.6)	0.8318	0.8319	0.8318	0.8319
JBG(a=k=0.6)	0.3409	0.3409	0.3409	0.3409
JBG(a=k=-0.6)	0.6590	0.6590	0.6590	0.6590
JBG(a=k=1.6)	0.1471	0.1470	0.1471	0.1470
JBG(a=k=-1.6)	0.8530	0.8529	0.8530	0.8529

From Table 3, we observe that, Bayes estimation with General Entropy loss function provides the highest AUC values when the loss parameter value is -1.6, according to the extension of Jeffreys prior value is 0.4 or 1.4.

So that the Bayes estimators of AUC under General Entropy loss function is best estimation method for Constant Shape Bi-Weibull Distribution using CHDAGE Data.

To demonstrate the proposed methodology with the help of graphical visualization, Figures 1 is drawn for comparing the Estimated AUC values under Constant Shape Bi-Weibull distribution using CHDAGE Data.

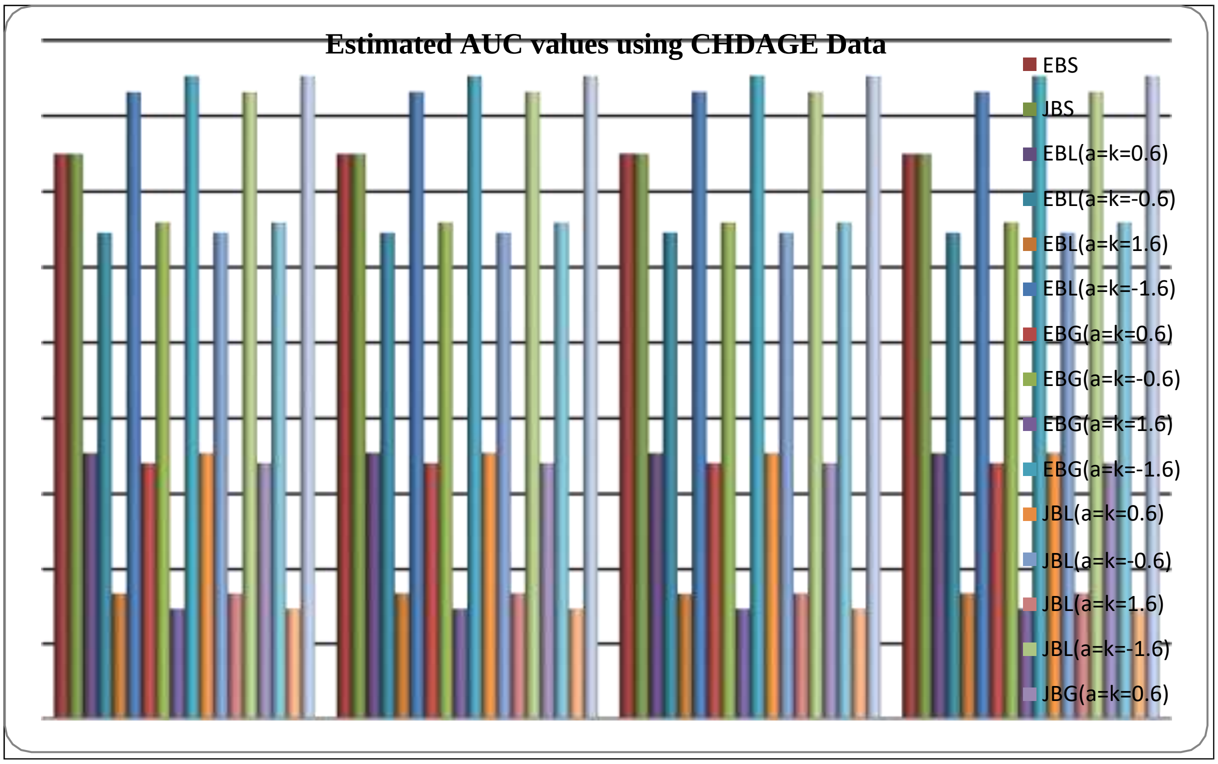

Figure 1 Effect on AUC values comparing different Estimation methods

From Figure 1, it is visualized that Bayes estimation with General Entropy loss function provides the highest AUC values when the loss parameter is -1.6.

5. CONCLUSION

The main objective for this paper is Bayesian estimation of AUC for the Constant Shape Bi-Weibull distribution, under three Loss functions, namely, the Linear Exponential (LINEX) Loss, General Entropy Loss, and Square Error Loss functions. Bayes estimators were obtained using Lindley approximation.

A Simulation study was conducted to examine and compare the performance of the estimators for different sample sizes with different values for the extension of Jeffreys‘ prior and Jeffreys‘ prior with the loss functions.

We also observe that Bayes estimation with General Entropy loss function provides the highest AUC values when the loss parameter is -1.6, according to the extension of Jeffreys prior value is 0.4 or 1.4. So that the Bayes estimators of AUC under General Entropy loss function is best estimation method for Constant Shape Bi-Weibull Distribution.

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