DOI:10.26524/jms.2017.8

Baye‟s estimation of stochastic inventory model for deteriorating items with trade credit

N.S.INDHUMATHY [1] AND DR. P.R.JAYASHREE [2]
RESEARCH SCHOLAR, DEPARTMENT OF STATISTICS, PRESIDENCY COLLEGE, CHENNAI-05
ASSISTANT PROFESSOR, DEPARTMENT OF STATISTICS, PRESIDENCY COLLEGE, CHENNAI-05

Abstract:
In this paper, a continuous time stochastic inventory model for deteriorating items with permissible delay in payments is considered. The demand is assumed to be a random variable with exponential distribution of time. In any classical inventory model, it was assumed that the purchaser must pay for the items received immediately. However, in practices, the supplier usually is willing to provide the purchaser a permissible delay of payments if the purchaser orders a large quantity. Hence, the focus is to find the optimal cycle time by maximising the profit function when a supplier provides a permissible delay of payments for a large order. However, the model contains the exponential parameter which is unknown and is estimated through MLE and Baye‘s under a squared error loss function. The conjugate Gamma prior is used as the prior distribution of exponential distribution. Finally, a numerical MCMC simulation is used to compare the estimators obtained with Expected risk and are shown graphically.

Key words: *Baye's estimator, Deterioration, Expected demand , Expected risk, Exponential distribution function, Permissible delay, Profit maximization, Optimal cycle time, , Squared error loss function*

1. INTRODUCTION

In Inventory policy, the organizations are taking serious attempt to manage inventories to get the competitive advantages over the other organization. Thus, the manager of every organization increasing their interest in optimizing the inventory decisions in a holistic way. As a result, they provide different types of facilities to others such as trade-credit. Many of the EOQ (economic order quantity) models have been widely used as a decision-making tool for managers to control inventory where payment patterns have changed under permit of delaying. It is understood that the retailers have to pay for the items as soon as the items are received. In practice, the supplier intends to raise his product‘s demand and so he will offer a delay period, namely, the trade credit period: Before the end of the trade credit period, retailer can sell the goods to accumulate revenue and earn interest. On the other hand, a higher interest is charged if the payment is not settled by the end of the trade credit period. . The permissible delay is an important source of financing for intermediate purchasers of goods and services. The permissible delay in payments reduces the buyer‘s cost of holding stock, because it reduces the amount of capital invested in stock for the duration of the permissible period. Thus it is a marketing strategy for the supplier is to attract new customers who consider it to be a type of price reduction.
In this regard, a number of research papers become visible which deal with the economic order quantity problem under the condition of permissible delay in payments. Goyal (1985) is

the first researcher to consider the economic order quantity (EOQ) inventory model under the condition of trade credit. Chand and Ward (1987) analyzed Goyal‗s model under assumptions of the ―classical‖ economic order quantity model and obtaining different results on trade credit. Shinn et al. (1996) extended Goyal (1985) paper by considering quantity discount for freight cost. Huang (2004) investigated that the unit purchasing price and the unit selling price are not necessarily equal within the EPQ framework under a supplier‗s trade credit policy. Teng et al. (2005) presented the optimal pricing and lot sizing model under permissible delay in payments by considering the difference between the purchase cost and the selling price and demand is a function of price. There are several relevant and interesting papers related to trade credit such as Chung et al. (2005), Mahata and Goswami (2007), Geetha K.V. and Uthayakumar R (2010), Liang-Yuh, *et.al.* (2006).
In this paper, a continuous time stochastic inventory model for deteriorating items with permissible delay in payments is considered. The aim is find the to find the optimal cycle period and optimal profit by maximising the profit function when a supplier provides a permissible delay of payments for a large order. However, the model contains the exponential parameter which is unknown and is estimated through MLE and Baye‗s under a squared error loss function.
Maximum likelihood estimation has been the widely used method to estimate the parameter of an exponential distribution. Lately Bayes method has begun to get the attention of researchers in the estimation procedure. The only statistical theory that combines modelling inherent uncertainty and statistical uncertainty is Bayesian statistics. There are several research papers available in the literature for Baye‗s estimation. Elli and Rao(1986) estimated the shape and scale parameters of the Weibull distribution. They minimized the corresponding expected loss with respect to a given posterior distribution. Sinha and Sloan (1988) obtained Bayes estimator of three parameters Weibull distribution. Chiou(1993) has given the Empirical Bayes shrinkage estimation on reliability in the exponential distribution. So, in this paper the conjugate Gamma prior is used as the prior distribution for parameter of exponential distribution and the parameter is also estimated through MLE. A numerical MCMC simulation is used to compare the estimators obtained with expected risk and are shown graphically.

2. ASSUMPTIONS AND NOTATIONS

The following assumptions are being made in this model

(1) Demand is a random variable with exponential distribution. The exponential probability density function with mean $1/\lambda$ is given by $f(t) = \lambda e^{-\lambda t}$ for $\lambda>0$.

(2) Shortages are not allowed and hence initial inventory level is equivalent to maximum demand level during the cycle period T.

(3) Items deteriorate with rate θ, $0 \leq \theta \leq 1$

The following notations are used in the model

c_1 : Selling cost per unit
c_2 : Ordering cost per unit
c_3 : Purchasing cost per unit
h : Carrying cost per unit
I_0 : Initial inventory level
I_e : The interest earned per rupee per year
I_p : The interest paid per rupee per year

1/λ : Expected rate of demand

θ : Rate of deterioration

TP : Total Profit

T : Cycle time

D : Permissible delay in settling account

3. DESCRIPTION OF THE MODEL

In this paper, the demand is assumed to be a random variable with exponential distribution of time and the average demand 1/λ. The non-instantaneous deterioration rate along with the demand function is considered. The average level of inventory in any cycle period T during which there is a demand is I_0 -1/λ and let the average rate of demand is T/λ. Hence expected average amount in inventory is

$$\int_0^T t.I_1(t) = \int_0^T \left\{ I_0 + \frac{\lambda}{\theta-\lambda}[1-e^{-\lambda t}]t \right\} dt \text{ ------- (1)}$$

$$\int_0^T t.\, I_1(t) = I_0 T + \frac{1}{\theta-\lambda}\left[\frac{e^{-\lambda T}(T\lambda+1)}{\lambda} + \frac{T^2\lambda^2-2}{2\lambda}\right] \text{---(2)}$$

The transition of inventory during the planning horizon can be represented by the following differential equation With the initial condition I_1 (0)=0

$$I_1'(t) = -\lambda e^{-\lambda t} - \theta I_1(t) \text{---------- (3)}$$

There are two distinct cases used in trade credit,

Case-I : Payment at or before the total depletion of inventory T>D

Case-II : Payment after depletion of inventory T≤D

CASE-I T>D

In this case the permissible payment time expires on or before the inventory depletion completely to zero. The total cost is comprised of the sum of ordering cost, carrying cost, and interest payable minus the interest earned. The items in stock are charged at interest rate I_p by the supplier starting at time D. Hence the interest payable per cycle for the inventory not sold after the period D is given by

$$c1I_p \int_D^T I_1(t)\, dt = c1I_P \int_D^T \left\{ I_0 + \frac{\lambda}{\theta-\lambda}[1-e^{-\lambda t}] \right\} dt \text{ ----------- (4)}$$

In equation (4) Truncated Taylor's series for exponential terms are,

$$e^{-\lambda T} = \frac{1-\lambda T+\lambda^2 T^2}{2}, \quad e^{-\lambda D} = \frac{1-\lambda D+\lambda^2 D^2}{2}$$

Interest payable is given by

$$c1I_P \int_D^T I_1(t)\, dt = \frac{c1I_P T}{\theta-\lambda}[T-D] \left[\frac{1-\lambda T+\lambda^2 T^2 - 1 + \lambda D - \lambda^2 D^2}{2\lambda} \right] \text{-------- (5)}$$

During the period of permissible delay, the buyer sells the product and the revenue from the sales used to earn interest. Therefore the interest earned during the positive inventory is given by

$$c_1 I_e \int_0^D \frac{1}{\lambda} t dt = \frac{c_1 I_e D^2}{2\lambda} \quad \text{-------} (6)$$

The total annual profit consists of the following

(a) Selling cost per unit per cycle = $\frac{c_1}{\lambda}$

(b) Ordering cost per unit time = $\frac{c_2}{T}$

(c) Purchasing cost per unit time = $\frac{c_3}{\lambda}$

(d) Carrying cost per unit time = $h\left\{\frac{T^2}{\lambda} + \frac{1}{\theta-\lambda}\left[\frac{\lambda^7 T^7 - 2\lambda^2 T^2}{2\lambda}\right]\right\}$

(e) Interest payable per cycle = $\frac{c1 I_p T}{\theta-\lambda}[T-D] \left[\frac{1-\lambda T+\lambda^2 T^2 - 1 + \lambda D - \lambda^2 D^2}{2\lambda}\right]$

(f) Interest earned per cycle = $\frac{c1 I_e D^2}{2\lambda}$

Therefore the total profit TP_1 is given by

$$TP_1 = \frac{c1}{\lambda} - \frac{c2}{T} - \frac{c3}{\lambda} - h\left\{\frac{T^2}{\lambda} + \frac{1}{\theta-\lambda}\left[\frac{\lambda^7 T^7 - 2\lambda^2 T^2}{2\lambda}\right]\right\}$$

$$-\left\{\frac{c1 I_p T}{\theta-\lambda}[T-D]\left[\frac{1-\lambda T+\lambda^2 T^2 - 1 + \lambda D - \lambda^2 D^2}{2\lambda}\right]\right\} - \frac{c1 I_e D^2}{2\lambda} \quad \text{-------}(7)$$

CASE- II T≤D

In this case customer sells all the items before expiration of permissible delay. There is no interest is paid only interest is earned on the given inventory. Hence the interest earned per year is,

$$c_1 I_e \left[\int_0^T \frac{1}{\lambda} t dt + \frac{1}{\lambda}(D-T)\right] = c1 I_e \left[\frac{T^2 + 2D - 2T}{2\lambda}\right] \text{---------}(8)$$

The total annual profit consists of the following

(a) Selling cost per unit per cycle = $\frac{c_1}{\lambda}$

(b) Ordering cost per unit time = $\frac{c_2}{T}$

(c) Purchasing cost per unit time = $\frac{c_3}{\lambda}$

(d) Carrying cost per unit time = $h\left\{\frac{T^2}{\lambda} + \frac{1}{\theta-\lambda}\left[\frac{\lambda^7 T^7 - 2\lambda^2 T^2}{2\lambda}\right]\right\}$

(e) Interest earned per cycle = $c1 I_e \left[\frac{T^2 + 2D - 2T}{2\lambda}\right]$

Total profit TP_2 is given by

$$TP_2 = \frac{c1}{\lambda} - \frac{c2}{T} - \frac{c3}{\lambda} - h\left\{\frac{T^2}{\lambda} + \frac{1}{\theta-\lambda}\left[\frac{\lambda^7 T^7 - 2\lambda^2 T^2}{2\lambda}\right]\right\} - c1 I_e\left[\frac{T^2 + 2D - 2T}{2\lambda}\right] \text{------ (9)}$$

Differentiating equation (7) with respect to T

$$\frac{\partial TP_1}{\partial T} = \frac{c2}{T^2} - h\left\{\frac{2T}{\lambda} + \frac{1}{\theta-\lambda}\left[\frac{7T^6\lambda^6 - 4\lambda T}{2}\right]\right\} - \frac{c1 I_p}{\theta-\lambda}\left[T\lambda - \frac{1}{2}\right] \text{------- (10)}$$

Again differentiating equation (10) with respect to T, we get

$$\frac{\partial^2 TP_1}{\partial T^2} < 0 \text{ ----(11)}$$

Differentiating equation (9) with respect to T

$$\frac{\partial TP_2}{\partial T} = \frac{c2}{T^2} - h\left\{\frac{2T}{\lambda} + \frac{1}{\theta-\lambda}\left[\frac{7T^6\lambda^6 - 4\lambda T}{2}\right]\right\} - c1 I_e \frac{1}{2\lambda}\left[2T - 2\right] \text{------- (12)}$$

$$\frac{\partial^2 TP_2}{\partial T^2} < 0 \text{ ---- (13)}$$

Hence the equations (11) and (13) are strictly concave functions

ALGORITHM

Step: 1 - For different values of cycle time T , determine TP_1 , TP_2 from (7) and (9) respectively.

Step: 2 - Repeat step (1), for all possible values of T with T>D until the maximum TP_1 is found from (7) and maximum TP_2 is found from (9).

Step: 3 - Calculate the optimal cycle TP^* , such that

$$TP^* = \max\begin{cases} TP_1^*, & \text{if } T > D \\ TP_2^*, & \text{if } T \le D \end{cases}$$

4. PARAMETER ESTIMATION

MAXIMUM LIKLIHOOD ESTIMATION

The probability density function of the exponential distribution is given by,

$$f(x\,;\lambda) = \begin{cases} \lambda e^{-\lambda x} & \text{if } x \ge 0 \\ 0 & \text{if } x < 0 \end{cases} \text{---------(1)}$$

Suppose $X_1, X_2, \ldots, X_n$ is a random sample from exponential distribution (1). Let $(x_1, x_2, \ldots, x_n)$ be the observed values of $(X_1, X_2, \ldots, X_n)$, then the likelihood function based on $(x_1, x_2, \ldots, x_n)$ is given by,

$$L(\lambda / X) = \prod_{i=1}^{n} f(x_i, \lambda) = \prod_{i=1}^{n} \lambda e^{-\lambda x} = \lambda^n e^{-\lambda \sum_{i=1}^{n} x_i}$$

To calculate the maximum likelihood estimator, the natural logarithm of likelihood function is maximised i.e. differentiating with respect to λ and equating each result to zero.

$$\frac{d}{d\lambda}\left(\ln L\left(\lambda / X\right)\right) = n \ln \lambda - \sum_{i=1} x_i = 0$$

The MLE of λ given by $\dfrac{n}{\sum_{i=1}^{n} x_i}$

BAYES ESTIMATION

In this section, we consider the Bayes estimation for the parameter λ assuming the conjugate of prior distribution for λ as two parameter Gamma distribution given as

$$f(\lambda/\alpha,\beta)=\begin{cases}\dfrac{\beta^{\alpha}}{\gamma(\alpha)}\lambda^{\alpha-1}e^{-\beta x} & ,\ \lambda\geq 0\\ 0 & ,\ \lambda<0\end{cases} \qquad \alpha>0,\ \beta>0$$

The likelihood function is assumed as L(λ/x) and the posterior distribution is,

$$p(\lambda/x)\propto L(\lambda/x)\, f(\alpha,\beta)$$

$$p(\lambda/X)\propto(\lambda^{n}e^{-\lambda\sum_{i=1}^{n}x_i})(\lambda^{\alpha-1}e^{-\beta})$$

$$p(\lambda/X)\propto\lambda^{n+\alpha-1}e^{-\lambda[\beta+\sum_{i=1}^{n}x_i]}$$

This follows Gamma distribution with parameter $\gamma(n+\tilde{\alpha},\ \tilde{\beta}+\sum_{i=1}^{n}x_i)$

The mean and variance are given by

$$\text{Mean}=\frac{\alpha}{\beta}=\frac{n+\tilde{\alpha}}{\tilde{\beta}+\sum_{i=1}^{n}x_i}$$

$$\text{Variance}=\frac{\tilde{\alpha}}{\tilde{\beta}^{2}}$$

5. NUMERICAL SIMULATION

To compare the different estimators of the parameter λ of the exponential distribution, the risks under squared error loss of the estimates are considered. These estimators are obtained by maximum likelihood and Bayes methods under Expected risk. The MCMC procedure for Baye‘s estimation is as follows

(i) A sample of size n is then generated from the density of the exponential distribution, which is considered to be the informative sample.

(ii) The MLE and Bayes estimators are calculated with $\alpha=n+\tilde{\alpha}$, $\beta=\tilde{\beta}+\sum_{i=1}^{n}x_i$

(iii) Steps (i) to (ii) are repeated N = 2000 times for different sample sizes and the risks under squared error loss of the estimates are computed by using;

Expected Risk $(\hat{\lambda})=\frac{1}{N}\sum_{i=1}^{n}\left(\hat{\lambda}_i-\lambda\right)^2$ Where, $\hat{\lambda}_i$ is the estimate at the i^{th} run

Assuming the value of λ = 0.001, the estimated value of $\hat{\lambda}$ using MLE and Baye‘s along with Expected risk are given in Table 1.

It is seen that for small sample sizes the estimators under the Expected Loss function have smaller ER when choosing proper parameters α and β. But for larger sample sizes (n>50) ,

all the estimators have approximately same ER. The obtained results are demonstrated in Table 1 and shown graphically in Figure 1 and 2. The estimated value of λ is $\hat{\lambda} = 0.0009$

TABLE-1: PARAMETER ESTIMATION AND EXPECTED RISK

n	Criteria	(i) **λ= 0.001,** $\alpha = n + \tilde{\alpha}$, $\beta = \tilde{\beta} + \sum_{i=1}^{n} x_i$			
		MLE	**α=0.5, β=0**	**α=1, β=0.5**	**α=1.5, β=1**
10	Estimated value ER	0.0017 0.000004	0.0018 0.0000006	0.0019 0.0000008	0.0020 0.0000009
25	Estimated value ER	0.0013 0.00000009	0.0013 0.00000009	0.0013 0.00000009	0.0014 0.00000009
50	Estimated value ER	0.0011 0.00000002	0.0011 0.00000002	0.0011 0.00000002	0.0011 0.00000002
75	Estimated value ER	0.0009 0.00000002	0.0009 0.00000002	0.0009 0.00000002	0.0009 0.00000002
100	Estimated value ER	0.0009 0.00000002	0.0009 0.00000002	0.0009 0.00000002	0.0009 0.00000002
125	Estimated value ER	0.0009 0.00000002	0.0009 0.00000002	0.0009 0.00000002	0.0009 0.00000002
150	Estimated value ER	0.0009 0.00000002	0.0009 0.00000002	0.0009 0.00000002	0.0009 0.00000002

FIGURE: 1 MLE AND BAYES ESTIMATION

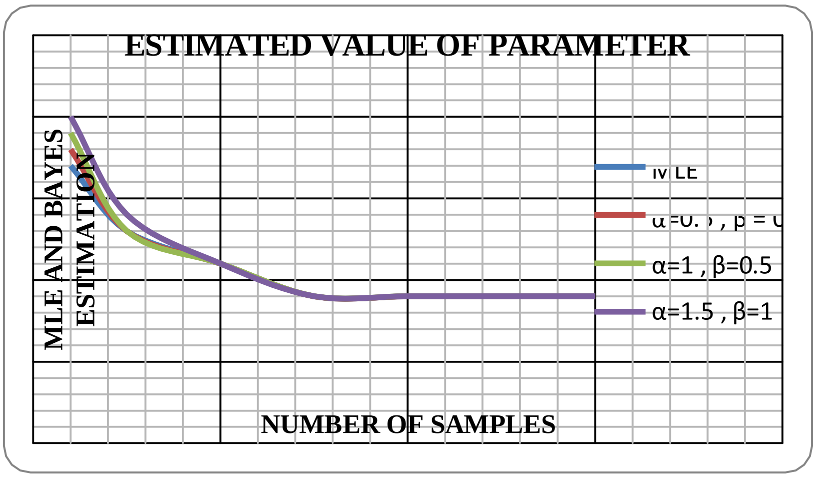
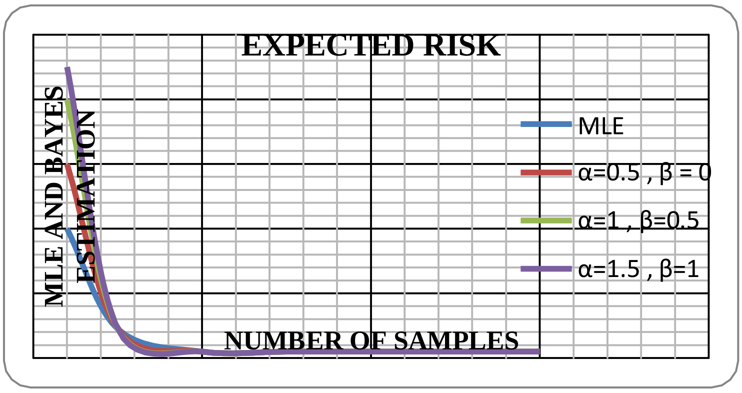


FIGURE: 2 EXPECTED RISK UNDER LOSS FUNCTION



6. NUMERICAL IIUSTRATION

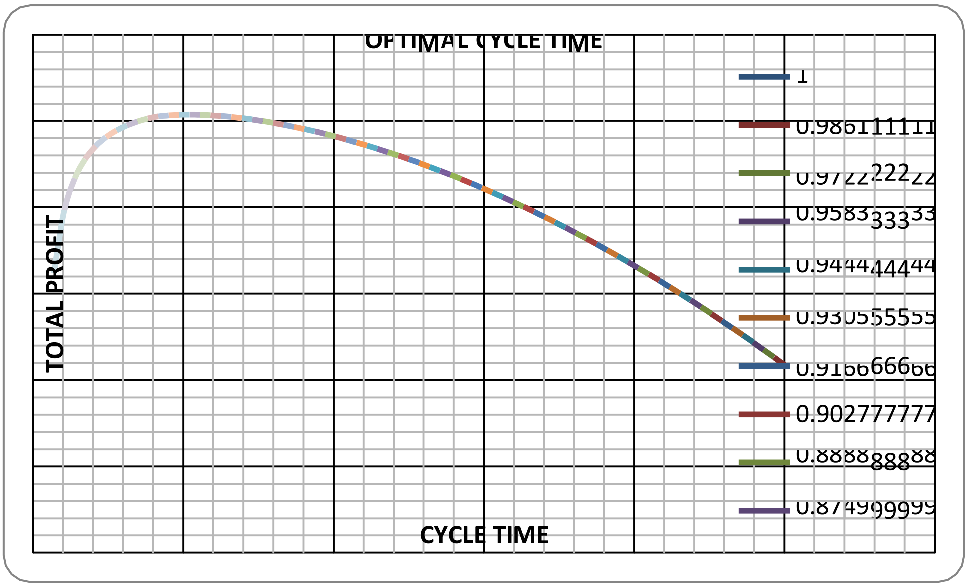
Using the estimated value of, $\hat{\lambda} = 0.0009$the optimal value of T is obtained by maximising total profit function by taking the values of other constants as D=60/360, c_1=20, c_2=60, c_3=15, θ=0.02, h=3, I_P=0.18 and I_e=0.16 . The R-software is used and the outputs for different trade credits are shown in Table 2. It is inferred that the maximum total profit value is 5073.558 in optimal cycle time 0.208333 years i.e., 75 days. The optimal cycle time with total profit is also shown graphically in Figure 2.

TABLE-2: VARIATION OF THE TOTAL PROFIT FOR DIFFERENT CYCLE TIME„T"

Case	CycleTime	TotalProfit
Case T > M	1	2175.409
Case T > M	0.986111	2263.61
Case T > M	0.972222	2350.592
Case T > M	0.958333	2436.352
Case T > M	0.944444	2520.888
Case T > M	0.930556	2604.197
Case T > M	0.916667	2686.277
Case T > M	0.902778	2767.124
Case T > M	0.888889	2846.737
Case T > M	0.875	2925.111
Case T > M	0.861111	3002.244
Case T > M	0.847222	3078.132
Case T > M	0.833333	3152.774
Case T > M	0.819444	3226.164
Case T > M	0.805556	3298.3
Case T > M	0.791667	3369.178
Case T > M	0.777778	3438.794
Case T > M	0.763889	3507.144
Case T > M	0.75	3574.224
Case T > M	0.736111	3640.029
Case T > M	0.722222	3704.556
Case T > M	0.708333	3767.798
Case T > M	0.694444	3829.752
Case T > M	0.680556	3890.411
Case T > M	0.666667	3949.77
Case T > M	0.652778	4007.823
Case T > M	0.638889	4064.562
Case T > M	0.625	4119.982
Case T > M	0.611111	4174.074
Case T > M	0.597222	4226.831
Case T > M	0.583333	4278.244
Case T > M	0.569444	4328.303
Case T > M	0.555556	4376.998
Case T > M	0.541667	4424.319

Case T > M	0.527778	4470.252
Case T > M	0.513889	4514.785
Case T > M	0.5	4557.904
Case T > M	0.486111	4599.592
Case T > M	0.472222	4639.831
Case T > M	0.458333	4678.601
Case T > M	0.444444	4715.881
Case T > M	0.430556	4751.646
Case T > M	0.416667	4785.868
Case T > M	0.402778	4818.515
Case T > M	0.388889	4849.552
Case T > M	0.375	4878.938
Case T > M	0.361111	4906.625
Case T > M	0.347222	4932.56
Case T > M	0.333333	4956.68
Case T > M	0.319444	4978.911
Case T > M	0.305556	4999.167
Case T > M	0.291667	5017.343
Case T > M	0.277778	5033.317
Case T > M	0.263889	5046.94
Case T > M	0.25	5058.029
Case T > M	0.236111	5066.361
Case T > M	0.222222	5071.655
Case T > M	**0.208333**	**5073.558**
Case T > M	0.194444	5071.612
Case T > M	0.180556	5065.225
Case T <= M	0.166667	5053.603
Case T <= M	0.152778	5028.118
Case T <= M	0.138889	4994.116
Case T <= M	0.125	4949.415
Case T <= M	0.111111	4890.742
Case T <= M	0.097222	4812.954
Case T <= M	0.083333	4707.48
Case T <= M	0.069444	4558.892
Case T <= M	0.055556	4336.331
Case T <= M	0.041667	3967.799
Case T <= M	0.027778	3237.294

FIGURE:3 TOTAL PROFIT FOR AN OPTIMAL CYCLE TIME



7. CONCLUSION

A continuous time stochastic inventory model for deteriorating items with permissible delay in payments is considered in this paper. The demand is assumed to be random variable with exponential time distribution. The MLE and Baye‘s estimation are used to estimate the parameter and by using the estimated value a numerical illustration is given for different parametric values. The optimal cycle time and optimal profit are found to be 75 days and Rs. 5073 (approximately) from the model.

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